Capstone projects 2025/26

Contents

1 Restrictions

- Theoretical Physics students can take any School of Mathematics TP or School of Physics project. They may take a Mathematics or Statistics project with agreement of the supervisor and Theoretical Physics course director, provided that it involves applications to physics. They may not take a project in Education in Mathematics, History of Mathematics, nor Philosophy of Statistics.

- Maths students can take any kind of project except the ones from the School of Physics that are listed in a separate document.

2 Projects in Mathematics

2.1 Nicholas Aidoo

Two projects in several complex variables.

2.2 Tommaso Cremaschi

Two projects around the following topics:

- Topology:

  - Classification of classes of 3-manifolds
  - Classification of surfaces both of finite and infinite-type
  - Elementary constructions of 4-manifolds
  - Knot Theory

- Geometry:

  - Counting geodesics in hyperbolic surfaces
  - Fuchsian groups and tessellations of the hyperbolic plane
  - Bass-Serre trees and group actions

2.3 Marvin Anas Hahn

Up to 2 projects in combinatorial algebraic geometry, depending on the student's background. Topics may include, e.g., Berkovich analytic spaces, Bruhat-Tits buildings, enumerative geometry and tropical geometry.

2.4 Adam Keilthy

2 projects in the general of algebraic geometry and number theory, including areas related to modular forms, polylogarithms, hyperplane arrangements and operadic structures. Some specific project ideas include

- Coalgebraic approaches to multiple zeta values

- Generalised operads from combinatorics and categories

- (Semi)group cohomology and deformations in algebra and number theory

though I am happy to discuss other options broadly in algebra and number theory based on student interests

2.5 Paschalis Karageorgis

2 projects related to either ordinary or partial differential equations. These will focus on recent research papers that involve applications of ODE/PDE in mathematical biology or physics. Students will be able to select the exact topic that they wish to study, but two typical suggestions are

- Epidemiology: `https://arxiv.org/pdf/2301.11947`

- Suspension bridge models `https://arxiv.org/pdf/2501.07940`

2.6 Miriam Logan

2 projects in the area of the history of Mathematics.

2.7 Nicolas Mascot

Up to 2 of the following:

- Project 1: Effective classification of quadratic forms over the rationals (reading+implementation)

  Prerequisite: Introduction to number theory

  The theory of quadratic forms over a field is a pleasant and satisfyingly complete area of number theory. Furthermore, all the proofs can be made completely constructive. For instance, it allows one to quickly see that the equation $x^2 + 2*y^2 = 7*z^2$ has no integer solutions apart from $(0,0,0)$. As another example, it leads us to discovering (not by inspection!) that $x = 5, y = 4, z = 3$ is a solution to $3*x^2 + 6*y^2 = 19*z^2$, from what we can deduce that every rational number is (explicitly) of the form $3*x^2 + 6*y^2 - 19*z^2$ for some rational x, y, z. The goal of this project is to gain an understanding of this theory, and to turn the proofs into algorithms which the student will then implement.

- Project 2: Elliptic and hyperelliptic curves in cryptography (reading+implementation)

  Prerequisite: Algebraic geometry (not strictly required)

  Elliptic curves are specific curves that come naturally equipped with a group law, which makes them amenable to cryptography. Hyperelliptic curves are a generalisation of elliptic curves; they are no longer equipped with a group law, but a construction of algebraic geometry, called curve Jacobians, makes it possible to recover a group law. The purpose of this project is to understand the group law of elliptic curves and of hyperelliptic curve Jacobians, and to implement them as well as some cryptography algorithms based on them.

- Project 3: Jacobians of compact Riemann surfaces and the Abel-Jacobi theorem (reading only)

  The purpose of this project is to understand the concept of curve Jacobians (cf. previous project), but this time over the complex numbers. This will lead us to proving the Abel-Jacobi theorem, a beautiful result (and beautiful proof!) in complex algebraic geometry.

Prerequisites: Introduction to complex analysis (required), Differential geometry (not strictly required)

2.8 Sergey Mozgovoy

Up to 2 projects:

- Project 1: Motivic classes of configuration spaces

  Getzler. Mixed Hodge structures of configuration spaces Gusein-Zade, Luengo, Melle-Hernández. Power structure over the Grothendieck ring of varieties and generating series of Hilbert schemes of points

  Prerequisites: Algebraic geometry, finite group representations, symmetric functions.

- Project 2: Multiple q-zeta values and quasi-shuffle algebras

  Bachmann, Kuehn. A short note on a conjecture of Okounkov about a q-analogue of multiple zeta values. Medina, Ebrahimi-Fard, Manchon. Unfolding the double shuffle structure of q-multiple zeta values.

2.9 Andreea Nicoara

2 projects in complex analysis (several complex variables), real algebraic geometry or model theory.

2.10 Nikhil Savale

Two projects in global geometric analysis. Topics may include –

- Spectral theory: Pseudodifferential calculus, Weyl's law and quantum ergodicity theorem.

- Index theory: the local index theorem, Morse inequalities via Witten's deformation.

- Complex geometry: holomorphic Morse inequalities and geometric quantization of compact Kahler manifolds via Toeplitz operators

- Sub-Riemannian geometry: Chow-Rashevskii and Ball-box theorems. Examples of abnormal geodesics.

2.11 Elizabeth Oldham

3 projects in the area of mathematics education. Projects may be based on, but are not restricted to, areas addressed in the module MAU34900 (and note that taking or having taken the module is not a prerequisite for choosing a mathematics education topic). Students are welcome to suggest topics of interest to them; however, these should be discussed with this supervisor to check for suitability and feasibility. The topics may involve theoretical study (based on literature in the field) or empirical research (collecting and analysing data, perhaps drawing on expertise in statistics). As undertaking empirical research usually involves obtaining ethical clearance, which can delay the work, it should be undertaken only after additional consultation.

2.12 Kirk Soodhalter

Up to two projects. Here are some general topics:

- Topics in applied linear algebra

- Analysis or algorithmic development in numerical linear algebra (iterative methods for linear systems and eigenvalue/vector approximation)

- Numerical PDEs (e.g., standard Finite Element methods)

- Signal and image reconstruction (applied analysis or algorithmic development)

- Operator Learning

- Willing to consider projects concerning numerical methods for computational modelling and simulation

2.13 John Stalker

Up to 3 projects:

- Project 1: The Peano Existence Theorem
  Content: There are two main ways of proving existence of solutions to the initial value problem for ordinary differential equations, due to Picard and Peano. Picard's method gives more information, including uniqueness and continuous dependence on initial conditions, at the expense of requiring stronger hypotheses. This project deals with Peano's method and the extent to which one can expect some version of continuous dependence on initial conditions under his hypotheses.
  Requires: Ordinary differential equations, advanced analysis.

- Project 2: Conservation Laws for Liquid Crystal Equations
  Content: The Oseen-Frank equations and Landau de Gennes equations describe equilibrium states of liquid crystals in the nematic phase. They are variational and have large numbers of symmetries. By Noether's theorem there are conservation laws corresponding to these symmetries. The project is to find those explicitly. Requires: Partial differential equations. Lie groups and differential geometry would be helpful.
  No actual physics is needed, but advanced classical mechanics might provide some useful intuition.

- Project 3: Gauss-Legendre Quadrature
  Content: There are a variety of methods for numerical integration. One of the more elegant ones is Gauss-Legendre quadrature, which approximates the integral of a function over an interval by a weighted sum of the values of the integrand at the roots of a Legendre polynomial. Since this is an approximation there is an error term, and the project focuses on the analysis of this error term. Requires: Real and complex analysis, ordinary differential equations. Functional analysis isn't strictly needed but might provide some helpful perspective.

- Project 4: Sparse Symmetric Linear Systems
  Content: Numerical solution methods for differential equations often lead to linear systems of equations which have a special structure: the coefficient matrices are symmetric and non-zero entries occur only on or near the diagonal. Luckily such systems can be solved much faster than general systems of the same size. There are fairly simple methods when the matrix is positive definite, which it often is in applications, and more complicated methods when the matrix is invertible but not positive definite.
  Requires: Linear algebra

- Project 5: The Lambda Calculus
  The lambda calculus is a theory of computation which actually predates computers but has played an important role in the design and implementation of programming languages, as well as having connections with mathematical logic. There are actually several directions this project could take, depending on the student's background and interests, including Church encodings, the Curry-Howard correspondence, and evaluation strategies. Some of these would include a programming element.
  Requires: Doesn't really depend on anything, but if you don't have at least some interest in mathematical logic or computer science you will probably not enjoy it.

- Persistent Data Structures
  Most programming problems require at least simple data structures like lists, queues, stacks or trees, and most programming languages provide at least some of them, either as a core part of the language or via libraries. In most cases the implementations are ephemeral. What I mean by this is that if you have, for example, a list and you modify it by changing an element somewhere in the middle then you no longer have access the old version. If you need both versions then you could of course create a new list which is the same as the old one except in that one position but the naive way of doing this, by copying the old list, uses time and memory proportional to the length of the list. Efficient data structures which allow access to past versions of the data are called persistent. There are a number of interesting ways of constructing persistent versions of commonly used data structures.
  Requires: Ability to program. The project could focus more on theory or on implementation but at least some implementation would be required.

2.14   Katrin Wendland

Up to 2 projects in complex geometry and/or modular forms (prerequisites: introduction to complex analysis, and in addition either elliptic functions and modular forms or differential geometry).

2.15   Dmitri Zaitsev

2 projects in complex analysis, mathematical finance, or applications of category theory to functional programming.

3 Projects in Statistics

3.1 Alessio Benavoli

Preference modelling lies at the intersection of economics, decision theory, machine learning and statistics. By learning individuals' preferences and how they make choices, we can build products/services that closely match their expectations, paving the way for more efficient and personalised applications across a wide range of domains. However,

Project: In many applications we do not observe preferences, but only the choices of an individual, that is the options the individual has selected among the given alternatives. Therefore, we must also be able to learn from choice data. A way to learning from choices is described here (Chapter 5) https://arxiv.org/pdf/2403.11782. This project aims to explore alternative models to learn from choices involving learning from sets instead of vectorised items as it is normally done in statistics and machine learning.

3.2 Cornelius Fritz

Up to four projects on:

- Project 1: Bayesian Manifold Learning
  In network modeling, edges are inherently interdependent: for instance, if you express interest in this project and contact me, it is more likely that a friend of yours, specifically one who shares similar interests and studies mathematics, will do the same. This interdependence complicates the statistical modeling of relational data, such as email communication networks. A widely used approach to address this complexity is the Latent Space Model (LSM), which assumes that each actor occupies a position in an unobserved latent space, and that the probability of a tie between any two actors is a function of the distance between their latent positions. There are multiple ways one could define this model. One could assume that the latent positions live in some specific space. Most commonly, the Euclidean space is used, where the distances are straight lines. In many cases, other geometries, such as the spherical or hyperbolic spaces, are possible, which all come with their definition of a distance. This Final Year Project will investigate Bayesian estimation techniques for latent space models in which positions are constrained to lie on a manifold. The Euclidean, spherical, and hyperbolic cases can all be viewed as specific instances of such manifolds. The project is designed to support two theses, one focusing on spherical and the other on hyperbolic geometry.

- Project 2: Choice of Latent Geometry
  In network modeling, edges are inherently interdependent: for instance, if you express interest in this project and contact me, it is more likely that a friend of yours, specifically one who shares similar interests and studies mathematics, will do the same. This interdependence complicates the statistical modeling of relational data, such as email communication networks. A widely used approach to address this complexity is the Latent Space Model (LSM), which assumes that each actor occupies a position in an unobserved latent space, and that the probability of a tie between any two actors is a function of the distance between their latent positions. There are multiple ways one could define this model: First, one could assume that the latent positions live in some specific space. Most commonly, the Euclidean space is used, where the distances are straight lines. In many cases, other geometries, such as the spherical or hyperbolic spaces, are possible, which all come with their definition of a distance. Second,

one can define alternative distances for set positions, such as the Manhattan or Chebyshev distances, even when positions remain Euclidean. This Final Year Project will investigate how the choice of geometry and distance metric affects the structure of networks generated under latent space models. The project will support two undergraduate theses.

3.3 Athanasios Georgiadis

- Project 1:
  Density estimation: We study the celebrated problem of estimation of a probability density function, of random variable distributed on metric spaces. The student may find more details, together with applications in Seismology, in the following paper.

- Project 2:
  Random Fields: We consider multivariate isotropic random fields on the ball of the Euclidean space. In this paper are studied properties such as regularity and approximation of such random fields and the study is accompanied with simulations.

- Project 3:
  Statistics in Business analytics: We study statistical approaches to problems of interest of Business analytics. The Students should see for example the paper Establishing the link: Does web traffic from various marketing channels influence direct traffic source purchases?, and then discuss with the supervisor on similar problems in the area.

3.4 James Ng

Up to three research projects are available in the areas of **spatially varying regression models** and **nonstationary spatial statistical modeling**:

- Bayesian varying regression modeling

  Luo, Z. T., Sang, H., & Mallick, B. K. (2021). *A Bayesian contiguous partitioning method for learning clustered latent variables in spatially varying regression models.* Journal of Machine Learning Research, **22**(37), 1–52.

  Li, F., & Sang, H. (2019). *Spatial homogeneity pursuit of regression coefficients for large datasets.* Journal of the American Statistical Association, **114**(527), 1050–1062.

- Scalable nonstationary spatial modeling

  Huang, H., et al. (2024). *Nonstationary spatial modeling of massive global satellite data.* Journal of Computational and Graphical Statistics.

3.5 Arthur White

2 projects among the following topics:

- Project 1: Bayesian statistics: a project in this area would be a good fit for students with an interest in technical and applied aspects of statistics. Specifically, this project

would investigate how to assess the impact of using different priors in a Bayesian data application. See for example: Morita, Satoshi, Peter F. Thall, and Peter Müller. "Determining the effective sample size of a parametric prior." Biometrics 64.2 (2008): 595-602. https://academic.oup.com/biometrics/article-abstract/64/2/595/7331626

- Project 2: Model based clustering: a project in this area would be a good fit for students interested in multivariate statistics and there will be an extra emphasis on computation, probably by implementing methods using R (or other software if the student has a preference.) Model based clustering methods are a formal way to cluster data: this could be multivariate and continuous, or categorical, or social network data. Some ideas: missing data imputation; variable selection; mixture of experts models (using additional variables to predict cluster membership); clustering data collected using stratified sampling schemes.

- Project 3: Modelling categorical/ordinal data. Students can focus on technical or applied elements of this project as they see fit. See for example: Hirk, Rainer, Kurt Hornik, and Laura Vana. "mvord: an R package for fitting multivariate ordinal regression models." Journal of Statistical Software 93 (2020): 1-41. https://www.jstatsoft.org/article/view/v093i04/0

3.6   Simon Wilson

- The photo-z problem

The Vera Rubin telescope is in Chile and has just begun testing. Once it is fully commissioned then it will provide unprecedented quantities of data on objects visible in the southern sky. One of its many goals is to more accurately estimate the distances of galaxies from us. It is expected to provide data on this for about 10 billion galaxies. However, there is a significant estimation problem in obtaining this distance from the data that the telescope provides. In this project we will look at that problem and the different statistical approaches to solving it, that usually revolve around estimating the red-shift of the galaxy, a quantity that is usually denoted by z.

3.7   Jason Wyse

Three projects in Bayesian methods, statistical computing, and/or applied statistics. Example topics might include, but are not limited to:

- Hamiltonian Monte Carlo (HMC) and Riemannian manifold HMC: framing a Bayesian estimation problem as a dynamical system.

- Probabilistic numerics: this is a way to quantify uncertainty in approximation of intractable integrals, a statistical approach to error quantification.

- Statistical interpretations of deep learning: how can currently popular deep learning approaches be viewed from a statistical perspective.

Here are some papers associated to the topics above:

Girolami, M. and Calderhead, B. (2011), Riemann manifold Langevin and Hamiltonian Monte Carlo methods. Journal of the Royal Statistical Society: Series B (Statistical Methodology), 73:

123-214.

Diaconis, P. (1988), Bayesian numerical analysis. Statistical decision theory and related topics IV, 163-175. Springer-Verlag, New York.

Nalisnick, E., Smyth, P. and Tran, D. (2023) A Brief Tour of Deep Learning from a Statistical Perspective. Annual Review of Statistics and Its Applications 10: 219-246.

4 Projects in Theoretical Physics, School of Mathematics

4.1 Ruth Britto

Up to 2 projects on mathematical properties of scattering amplitudes in quantum field theory. General references: `https://arxiv.org/abs/2201.03593`, `https://arxiv.org/abs/2306.05976`. Sample topics:

- Curve integrals for amplitudes [see `https://arxiv.org/abs/2408.11891`]

- Cosmological correlators [see `https://arxiv.org/abs/2411.09695`]

- Polytopes, positive geometries and singularities. [see `https://arxiv.org/abs/1412.8478`, `https://arxiv.org/abs/1611.08325`]

4.2 Marius De Leeuw

One project on "Temperley-Lieb and Fuss-Catalan algebras"

4.3 Patrick Fritzsch

Up to 2 projects related to Monte Carlo simulations in theoretical physics, for instance:

- A theoretical and numerical investigation of the work ""A Statistical Approach to Quantum Mechanics"", by M. Creutz and B. Freedman, using modern simulation techniques. Reference

- Investigation of phase transitions and critical exponents in field theories.

- Another topic in Lattice Field Theory

4.4 Sergey Frolov

2 reading projects. Students taking these projects must also take Differential Geometry, and Quantum Field Theory.

- Project 1 (Difficulty: Medium-High): "Integrable Systems".
  A student will read the book by Gleb Arutyunov , "Elements of Classical and Quantum Integrable Systems", `https://link.springer.com/book/10.1007/978-3-030-24198-8`

- Project 2 (Difficulty: Medium-High): "Supersymmetric Quantum Mechanics and Equivariant Localisation".
  A student will read the lectures by David Tong "Supersymmetric Quantum Mechanics", `http://www.damtp.cam.ac.uk/user/tong/susyqm.html` and the lectures by Richard J. Szabo "Equivariant Localization of Path Integrals", `https://arxiv.org/pdf/hep-th/9608068`

4.5 Tristan McLoughlin

Two projects:

- Project 1:
  Title: Self-dual gravity and integrability

  Abstract: Self-dual gravity is a truncation of Einstein's theory of gravity in which the self-dual part of the Weyl tensor is vanishing. This theory is of interest as a toy model because of its simplicity, its good quantum behaviour and its connection to the perturbative computation of graviton scattering amplitudes. It is also known to be classically integrable and has an elegant description in terms of twistor theory. In this project we will review the construction of this theory, the derivation of Plebanski's "heavenly" equations, and the corresponding Lax operators. We will then study the construction of classical solutions by means of the dressing procedure and related methods.

  References:

  J. F. Plebanski, "Some solutions of complex Einstein equations," J. Math. Phys. 16 (1975), 2395-2402

  M. Dunajski, L.J. Mason, and N. Woodhouse, "From 2D integrable systems to self-dual gravity," Journal of Physics A: Mathematical and General 31 (1998), 6019-6028

  K. Krasnov, "Self-Dual Gravity", Class.Quant.Grav. 34 (2017) 9, 095001, eprint: 1610.01457 [hep-th]

- Project 2:
  Title: Long-range spin-chains and entanglement entropy

  Abstract: Quantum spin-chains are a paradigmatic model for quantum many-body systems. In one-dimension certain spin-chains are known to be integrable and analytical results are available by methods such as the algebraic Bethe ansatz. Spin-chains with long-range interactions, while of interest in many physical systems, are less well studied and there are fewer analytical tools. In this project we will study specific models, in the particular the so-called Inozemtsev model, which are known to be integrable but have all-to-all interactions. One goal will be to numerically study these models, their spectrum, eigenstate entropy and if possible measures of entanglement such as bi-partite entanglement entropy and the entanglement spectrum.

  References:

  V. I. Inozemtsev, On the connection between the one-dimensional $s = 1/2$ Heisenberg chain and Haldane–Shastry model, J. Stat. Phys. 59 (1990), no. 5-6, 1143–1155.

  R. Klabbers and J. Lamers, The deformed Inozemtsev spin chain, SciPost Phys. 17 (2024), 155, eprint: arXiv:2306.13066.

A. Gubin, L.F. Santos, Quantum chaos: An introduction via chains of interacting spins 1/2
American Journal of Physics 80 (3), 246-251

4.6   Jan Manschot

1 or 2 projects on the following subjects:

- Project 1: The Dirac equation on 4-manifolds (prerequisite modules: MAU34409 Applied Differential Geometry and MAU44400 Quantum Field Theory)

- Project 2: Quantum Black Holes and 2-dimensional Conformal Field Theory (prerequisite modules: MAU44404 General Relativity and MAU44400 Quantum Field Theory)

4.7   Andrei Parnachev

Up to 2 projects on "Theoretical developments in cosmological inflation".
Prerequisites: The students must be taking MAU44400 and MAU44404 concurrently.
Preliminary content: Cosmological inflation, cosmic microwave background, quantum field theory and other constraints on inflationary models.

4.8   Manya Sahni

- Project 1:
  Black hole thermodynamics and black hole entropy - this project will study the origin of the laws of black hole mechanics and entropy of black holes arising from the interaction of quantum particles with classical black holes. Further topics can include Hawking radiation, Wald Entropy of a black hole as a Noether charge, black hole information.
  This project would suit a student taking QFT, GR and Differential geometry.

- Project 2:
  Supersymmetry: we will begin with a brief overview of spinors in quantum field theory, followed by the basics of supersymmetry. Depending on the student's progress, we will also review a method to perform exact calculations in quantum field theory using a technique called supersymmetric localisation.
  This project would suit a student taking QFT.

4.9   Stefan Sint

Two projects:

- Topic 1: Confinement in SU(3) pure gauge theory

  This topic requires to first study non-Abelian gauge symmetries in the continuum before moving to a space-time lattice formulation, where the confinement criterion by Wilson can be studied both in a strong coupling expansion and by measuring Wilson loops in a numerical simulation.

  Refs:

  1) QFT textbooks, e.g. Schroeder & Peskin

  2) K. Wilson's classic paper: "Confinement of quarks", Phys.Rev.D 10 (1974) 2445

- Topic 2: Numerical stochastic perturbation theory.

  Treating interactions in a Quantum Field Theory can be done in various ways. If the coupling is small, a perturbative expansion to low orders can be done in terms of Feynman diagrams. An alternative is based on stochastic quantization. It is proposed to use the technique to obtain the perturbative expansion of a scalar ϕ^4 theory on a space time lattice. This could be compared to a non-perturbative simulation in order to assess the accuracy.

  Refs:

  1) "Numerical stochastic perturbation theory for full QCD" F. Di Renzo, L. Scorzato hep-lat/0410010 [hep-lat] DOI: 10.1088/1126-6708/2004/10/073 Published in: JHEP 10 (2004), 073

  2) "SMD-based numerical stochastic perturbation theory" M. Dalla Brida and M. Luescher, Eur.Phys.J.C 77 (2017) 5, 308 1703.04396 [hep-lat]

  3) "Investigation of New Methods for Numerical Stochastic Perturbation Theory in phi^4-Theory" M. Dalla Brida, M. Garofalo, A.D. Kennedy e-Print: 1703.04406 [hep-lat] DOI: 10.1103/PhysRevD.96.054502 Phys.Rev.D 96 (2017) 5, 054502

4.10 Chaolun Wu

Up to 2 projects. Topics include:

- Aspects of magnetic monopoles in Yang-Mills theory in relation to the dual superconductivity picture of confinement, based on the Cho-Faddeev-Niemi decomposition (for the Cho decomposition, see, for example, `https://arxiv.org/abs/hep-th/0201179` and `https://arxiv.org/abs/1503.08890`, but the actual content of the project to be explored will be different than these papers).

- Worldline path integral formalism of quantum field theory (partially based on `https://indico.cern.ch/event/206621/attachments/317310/442802/LecturesMorelia_OC.pdf` and `https://indico.cern.ch/event/206621/attachments/317309/442801/lectures_morelia_CS.pdf`)

- Thermal quantum field theory and effective action for dissipative hydrodynamics (partially based on `https://arxiv.org/abs/1805.09331`)

- Classification of topological insulators and superconductors (see `https://arxiv.org/abs/0912.2157` to get a taste; taking MAU34407 is preferred, but not required)

- Structure of Landau quasiparticle in Fermi liquid: conventional quantum field theoretical approach to interacting electrons in metal is based on Feynman diagrams/Dyson series of the electron. Here we explore the possibility of a different approach based on change of variable to the quasiparticle at the field/operator level.

Students are welcome to contact this supervisor in advance to discuss the specifics.